



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS:

Lecture 16: Pile Foundations under Lateral Loads - Elastic Method for Cohesionless Soils

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ERD of Foundations - Modules

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Module - 3

Pile Foundations



Module -3: Chapters

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Chapter 2: Lateral Loads on Piles

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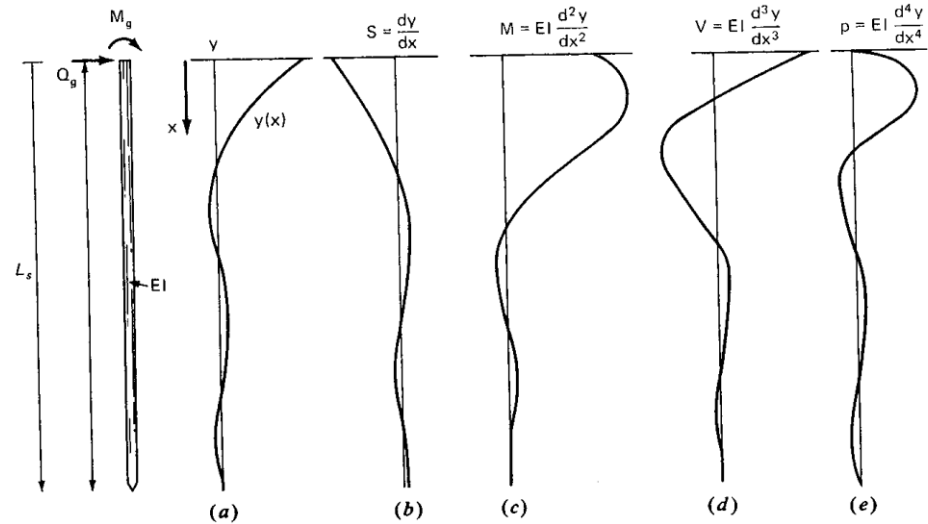


Solution for Piles in Cohesionless Soil

A typical pile of length L_s and stiffness EI is shown in Fig. The depth x is measured downward from the ground line. The boundary conditions at the top consist of an imposed moment M_g and a shear Q_g and each is shown acting in a positive sense. The soil modulus variation for which the solution is available as

$$k = \eta_h x \quad (0)$$

Fig. a shows the $y(x)$ curve of Pile. Once this is known, its derivatives yield slope moment, shear and soil reaction are shown in Fig. b, c, d and e respectively.



Solution for Piles in Cohesionless Soil (Continued...)

Relative stiffness factor, $(T) = \frac{L\alpha}{\lambda}$ (1)

λ = positive integer, $L\alpha$ = Infinite length of pile(depends upon EI of pile). Now $y(x)$ may be expressed as

$$y(x) = f(x, T, L_s, k, EI, Q_g, M_g) \quad (2)$$

If deflections are small and if an elastic behaviour is assumed, the principal of superposition may be applied.

If y_A represents the deflection caused by the lateral load Q_g and y_B the deflection caused by M_g , the total deflection is given by

$$y = y_A + y_B \quad (3)$$

$$\frac{y_A}{Q_g} = f_A(x, T, L_s, k, EI) \quad (4a)$$

$$\frac{y_B}{M_g} = f_B(x, T, L_s, k, EI) \quad (4b)$$

Solution for Piles in Cohesionless Soil (Continued...)

Writing Equation 4 in dimensionless form

$$\text{Case A:} \quad \frac{y_A EI}{Q_g T^3}, \frac{x}{T}, \frac{L_s}{T}, \frac{k T^4}{EI} \quad (5a)$$

$$\text{Case B:} \quad \frac{y_b EI}{M_g T^2}, \frac{x}{T}, \frac{L_s}{T}, \frac{k T^4}{EI} \quad (5b)$$

$$\text{Where, } \frac{x}{T} = Z \text{ (depth coefficient)} \quad (6a)$$

$$\frac{L_s}{T} = Z_{max} \text{ (maximum depth coefficient)} \quad (6b)$$

$$\frac{k T^4}{EI} = \phi(z) \text{ (Soil modulus function)} \quad (6c)$$

$$\frac{y_A EI}{Q_g T^3} = A_y \text{ (deflection coefficient for case A)} \quad (6d)$$

$$\frac{y_b EI}{M_g T^2} = B_y \text{ (deflection coefficient for case B)} \quad (6e)$$

Solution for Piles in Cohesionless Soil (Continued...)

Thus from equation (6d) and (6e)

Total deflection, $y_x = y_A + y_B = A_y \frac{Q_g T^3}{EI} + B_y \frac{M_g T^2}{EI}$ (7a)

Slope, $S_x = S_A + S_B = A_s \frac{Q_g T^2}{EI} + B_s \frac{M_g T}{EI}$ (7b)

Moment, $M_x = M_A + M_B = A_m Q_g T + B_m M_g$ (7c)

Shear, $V_x = V_A + V_B = A_v Q_g + B_v \frac{M_g}{T}$ (7d)

Soil Reaction, $p_x = p_A + p_B = A_p \frac{Q_g}{T} + B_p \frac{M_g}{T^2}$ (7e)

From the theory of beam on elastic foundation, the equation of deflected beam is

$$EI \frac{d^4 y}{dx^4} = p \quad (8a)$$

Solution for Piles in Cohesionless Soil (Continued...)

However, in Winkler Hypothesis $p = -ky$

Therefore,
$$\frac{d^4 y}{dx^4} + \frac{k}{EI} y = 0 \quad (8b)$$

For Case A:
$$\frac{d^4 y_A}{dx^4} + \frac{k}{EI} y_A = 0 \quad (9a)$$

For Case B:
$$\frac{d^4 y_B}{dx^4} + \frac{k}{EI} y_B = 0 \quad (9b)$$

Substituting for y_A , $\frac{k}{EI}$ and $\frac{x}{T}$ in nondimensional form from Eqn. (6d), (6c) and (6a), respectively.

For Case A:
$$\frac{d^4 A_y}{dz^4} + \phi(z) A_y = 0 \quad (10a)$$

For Case B:
$$\frac{d^4 B_y}{dz^4} + \phi(z) B_y = 0 \quad (10b)$$

Solution for Piles in Cohesionless Soil (Continued...)

For solution of equation

Substituting for y_A , $\frac{k}{EI}$ and $\frac{x}{T}$ in nondimensional form from Eqn. (6d), (6c) and (6a), respectively.

For Case A:
$$\frac{d^4 A_y}{dz^4} + \phi(z) A_y = 0 \quad (10a)$$

For Case B:
$$\frac{d^4 B_y}{dz^4} + \phi(z) B_y = 0 \quad (10b)$$

For solution of Eqn. (10a) and (10b) it is necessary to specify $\phi(z)$ and to define T.

For sands ($k = \eta_h x$) and other soils, where soil modulus (k) may be assumed to linearly increase with depth. $\phi(z)$ may be equated to z .

$$\phi(z) = z \quad (11a)$$

Solution for Piles in Cohesionless Soil (Continued...)

$$\frac{kT^4}{EI} = \frac{\eta_h x T^4}{EI} = \frac{x}{T} \quad (11b)$$

$$T = \left(\frac{EI}{\eta_h} \right)^{1/5} \quad (12)$$

Reese and Matlock (1956) used the finite difference method for equating the coefficient A and B for finding the deflection (y), slope (S), moment (M), shear (V) and soil reaction (p) coefficient Q_g and M_g .

- Piles with $Z_{max} \leq 2$, pile behave as a rigid pile. A_y and B_y linearly decrease with depth.
- Piles with $Z_{max} \geq 5$, A_y and B_y are more or less same and thus pile length beyond $Z_{max} = 5$ is not effective in altering the deflection, therefore infinite length.

$$L_\alpha = 5T \quad (13)$$

Solution for Piles in Cohesionless Soil (Continued...)

Below Table provide the coefficient A and B. which is applicable when $Z_{max} \geq 5$

Coefficient, A for long pile

Z	A_y	A_S	A_M	A_V	A_p
0.0	2.435	- 1.623	0.000	1.000	0.000
0.1	2.273	- 1.618	0.100	0.989	- 0.227
0.2	2.112	- 1.603	0.198	0.956	- 0.422
0.3	1.952	- 1.578	0.291	0.906	- 0.586
0.4	1.796	- 1.545	0.379	0.840	- 0.718
0.5	1.644	- 1.503	0.459	0.764	- 0.822
0.6	1.496	- 1.454	0.532	0.677	- 0.897
0.7	1.353	- 1.397	0.595	0.585	- 0.947
0.8	1.216	- 1.335	0.649	0.489	- 0.973
0.9	1.086	- 1.268	0.693	0.392	- 0.977
1.0	0.962	- 1.197	0.727	0.295	- 0.962
1.2	0.738	- 1.047	0.767	0.109	- 0.885
1.4	0.544	- 0.893	0.772	- 0.056	- 0.761
1.6	0.381	- 0.741	0.746	- 0.193	- 0.609
1.8	0.247	- 0.596	0.696	- 0.298	- 0.445
2.0	0.142	- 0.464	0.628	- 0.371	- 0.283
3.0	- 0.075	- 0.040	0.225	- 0.349	0.226
4.0	- 0.050	0.052	0.000	- 0.106	0.201
5.0	- 0.009	0.025	- 0.033	0.013	0.046

Coefficient, B for long pile

Z	B_y	B_S	B_M	B_V	B_p
0.0	1.623	- 1.750	1.000	0.000	0.000
0.1	1.453	- 1.650	1.000	- 0.007	- 0.145
0.2	1.293	- 1.550	0.999	- 0.028	- 0.259
0.3	1.143	- 1.450	0.994	- 0.058	- 0.343
0.4	1.003	- 1.351	0.987	- 0.095	- 0.401
0.5	0.873	- 1.253	0.976	- 0.137	- 0.436
0.6	0.752	- 1.156	0.960	- 0.181	- 0.451
0.7	0.642	- 1.061	0.939	- 0.226	- 0.449
0.8	0.540	- 0.968	0.914	- 0.270	- 0.432
0.9	0.448	- 0.878	0.885	- 0.312	- 0.403
1.0	0.364	- 0.792	0.852	- 0.350	- 0.364
1.2	0.223	- 0.629	0.775	- 0.414	- 0.268
1.4	0.112	- 0.482	0.688	- 0.456	- 0.157
1.6	0.029	- 0.354	0.594	- 0.477	- 0.047
1.8	- 0.030	- 0.245	0.498	- 0.476	0.054
2.0	- 0.070	- 0.155	0.404	- 0.456	0.140
3.0	- 0.089	0.057	0.059	- 0.213	0.268
4.0	- 0.028	0.049	- 0.042	0.017	0.112
5.0	0.000	0.011	- 0.026	0.029	- 0.002

Solution for Piles in Cohesionless Soil (Continued...)

Now two cases are: (1) Q_g is acting (2) M_g is acting

Let us consider Q_g is acting. However, pile head is restrained against rotation ($S_o = 0$)

For Slope,

$$S_o = S_A + S_B = A_s \frac{Q_g T^2}{EI} + B_s \frac{M_g T}{EI} = 0 \quad \text{From Table} \quad (14)$$

$$\frac{M_g}{Q_g T} = \left(\frac{A_s}{B_s} \right)_{z=0} = - \left(\frac{-1.623}{-1.750} \right) = -0.93 \quad (15a)$$

$$M_g = -0.93 Q_g T \quad (15b)$$

Solution for Piles in Cohesionless Soil (Continued...)

For deflection,

$$y = y_A + y_B = A_y \frac{Q_g T^3}{EI} + B_y \frac{M_g T^2}{EI}$$
$$y = (A_y - 0.93 B_y) \frac{Q_g T^3}{EI} \quad (16a)$$

$$y = (C_y) \frac{Q_g T^3}{EI} \quad (16b)$$

Where, $C_y = A_y - 0.93 B_y$ (16c)

For partial fixity, $C_y = (A_y - 0.93 \lambda B_y)$ (16d)

$\lambda = 0$ for free head

$\lambda = 1$ for fixed head

$\lambda = 0.5$ for 50% restrained

For moment,

$$C_m = A_m - 0.93 B_m \quad (17a)$$

$$M = C_m Q_g T \quad (17b)$$

$$C_m = A_m - 0.93 \lambda B_m \quad (17c)$$

Solution for Piles in Cohesionless Soil (Continued...)

For soil reaction,

$$C_p = A_p - 0.93B_p \quad (18a)$$

$$p = C_p \frac{Q_g}{T} \quad (18b)$$

or

$$C_p = A_p - 0.93\lambda B_p \quad (18c)$$

References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Ranjan G. and Rao A.S.R. (2004), "Basic and Applied Soil Mechanics", New Age Int. Ltd., New Delhi.**

Thank You

